

# Practice Problem 1

Prove that the expression

$$\frac{\gcd(m,n)}{n} \binom{n}{m}$$

is an integer for all pairs of integers  $n \geq m \geq 1$ .

*Solution:*  $\gcd(m,n)$  can be written as an integer linear combination of  $m$  and  $n$ , that is, for some integers  $k, l$ , we have  $km + ln = \gcd(m,n)$ . It follows that

$$\frac{\gcd(m,n)}{n} \binom{n}{m} = \frac{km + ln}{n} \binom{n}{m} = k \frac{m}{n} \binom{n}{m} + l \frac{n}{n} \binom{n}{m} = k \binom{n-1}{m-1} + l \binom{n}{m}$$

which is an integer.