

TRU Math Competition Practice Problem 1

Deadline: October 8, 2018

Let f be a continuous function on $[0, 1]$. Show that if $-1 \leq f(x) \leq 1$ for all $x \in [0, 1]$, then there is $c \in [0, 1]$ such that $[f(c)]^2 = c$.

Solution: Define $g(x) = f^2(x) - x$. It is a continuous function on $[0, 1]$ and since $g(0) = f^2(0) \geq 0$ and $g(1) = f^2(1) - 1 \leq 0$, by the Intermediate Value Theorem there is a $c \in [0, 1]$ such that $g(c) = 0$. That is, for some $c \in [0, 1]$ we have $f^2(c) = c$.