

TRU Math Competition Practice Problem 5

Deadline: November 5, 2018

Suppose that A and B are two square matrices of the same size for which the indicated inverses exist. Prove that

$$(A + AB^{-1}A)^{-1} + (A + B)^{-1} = A^{-1}.$$

(University of Toronto, Undergraduate Mathematics Competition)

Solution:

$$\begin{aligned}(A + AB^{-1}A)^{-1} + (A + B)^{-1} &= (A(I + B^{-1}A))^{-1} + (A + B)^{-1} \\ &= (AB^{-1}(B + A))^{-1} + (A + B)^{-1} \\ &= (B + A)^{-1}BA^{-1} + (A + B)^{-1} \\ &= (A + B)^{-1}(BA^{-1} + I) \\ &= (A + B)^{-1}(B + A)A^{-1} \\ &= A^{-1}\end{aligned}$$