

TRU Math Competition Practice Problem 6

Deadline: November 12, 2018

Let A, B , and M be $n \times n$ matrices with real entries where M is invertible and $AM = MB$. Prove that $\det(A - MX) = \det(B - XM)$ for every $n \times n$ matrix X with real entries.

Solution:

$$\begin{aligned}\det(A - MX) &= \det(M^{-1}) \det(A - MX) \det(M) \\ &= \det(M^{-1} \underbrace{AM}_{=MB} - \underbrace{M^{-1}M}_{=I} XM) \\ &= \det(M^{-1}MB - XM) \\ &= \det(B - XM).\end{aligned}$$