TRU I Math Battle, 2022

To not know math is a severe limitation to understanding the world

Richard P. Feynman

1. Find all possible natural numbers x, y, z such that $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} = 1$.

2. Solve the system of equations $\begin{cases} x + y^2 = y^3 \\ y + x^2 = x^3 \end{cases}$

3. Find last 3 digits of $1^{2022} + 2^{2022} + 3^{2022} + \ldots + 999999^{2022}$

4. A square $ABCD$ with sides of length 1 is given(Fig.1(a)). On its diagonal AC and side the CD the equilateral triangles ACM and CDN are constructed. What is the length of MN?

5. Find all triples (a, b, c) of consecutive natural numbers for which the sum of their three paired products $(ab + ac + bc)$ is equal to 3×10^k, where k – some natural number.

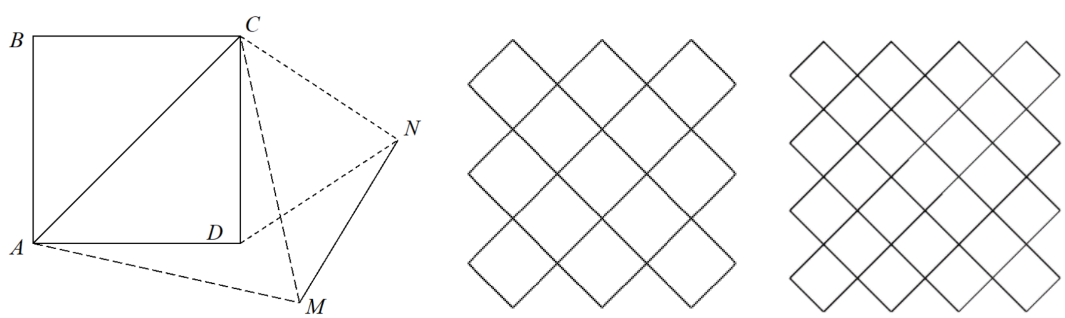
6. In the parallelogram $ABCD$, the sides AB, BC, CD, AD and diagonals AC and BD are painted either yellow or blue such that there are 3 segments of each color. Prove that you can always construct a triangle from segments of one of the colors, that is a triangle that is either only blue or yellow.

7. You have a card with the number 12 on it. You can add a card to your collection of cards by the following conditions:

   - if you have a card with a number n, then you can add the card with the number $2n + 1$;
   - if you have a card with a number m, which is is divisible by 3, you can add the card $m/3$.

   Can you get a card with the number 29? What about 100?

8. Consider figures consisting of single squares, which are enclosed in the way shown on the Fig.2(b) and Fig.2(c). Prove that the number of small squares in each such figure (not only in the given one, but also in any similar) is equal to the sum of the squares of any two natural numbers.



(a) (b) (c)

Figure 1: (a) Figure 1 (b) Figure 2.1 (c) Figure 2.2

Organized by Kateryna Tretiakova